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Non-Gaussianity from Inflation

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Abstract

Correlated adiabatic and isocurvature perturbation modes are produced during inflation through an oscillation mechanism when extra scalar degrees of freedom -other than the inflaton field are present. We show that this correlation generically leads to sizeable non-Gaussian features both in the adiabatic and isocurvature perturbations. The non-Gaussianity is first generated by large non-linearities in some scalar sector and then efficiently transferred to the inflaton sector by the oscillation process. We compute the cosmic microwave background angular bispectrum, providing a characteristic feature of such inflationary non-Gaussianity, which might be detected by upcoming satellite experiments.

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1 Introduction

It is generally believed that inflation provides the causal mechanism to seed structure formation in the Universe. One of the most interesting aspects of these primordial perturbations is their statistical nature. The simplest and most generally accepted idea is that these primordial perturbations were Gaussian distributed. However, this issue is far from being settled: there is still ample room for some level of non-Gaussianity in the initial conditions.

One way of parametrizing the possible presence of non-Gaussianity in the primordial gravitational potential Φ is to expand it in the following way [1, 2, 3, 4]

$$\Phi = \varphi + f_{\rm NL}(\varphi^2 - \langle\varphi^2\rangle) + \mathcal{O}\left(f_{\rm NL}^2\right) \ , \tag{1}$$

where φ is a zero-mean Gaussian random field and $f_{\rm NL}$ is an expansion parameter which can be observationally constrained.

It is commonly believed that primordial perturbations generated during inflation are necessarily adiabatic and Gaussian. Although this is essentially the case for the simplest model, where a single inflaton field undergoes a slow-roll transition, the range of possibilities is actually much wider and more interesting than such a standard lore may tell. Even in the case of a single, slowly rolling inflaton field, it has been shown that the effect of field non-linearities and their backreaction on the underlying geometry is to generate a small, but calculable, non-Gaussianity [2, 5, 6, 7]. The non-Gaussianity, or non-linearity, parameter $f_{\rm NL}$ can be expressed in terms of the standard slow-roll parameters ϵ and η as $f_{\rm NL} \sim 3\epsilon - 2\eta$ [5, 6, 7]. Since the slow-roll parameters have to be much smaller than unity for inflation to occur, the typical value of $f_{\rm NL}$ in single-field inflationary models is inevitably tiny. These constraints can be partially relaxed if the inflaton potential contains 'features' in that part corresponding to the last ~ 60 e-foldings [8, 9, 6].

The common belief that non-Gaussianity of inflation generated perturbations is small comes from this theoretical argument applied to single-field models of inflation. On the other hand the presence of non-Gaussianity is only mildly constrained by observations. Let us focus on the evidence for Gaussian primordial fluctuations coming from the analysis of primary anisotropies of the Cosmic Microwave Background (CMB), as these certainly provide the most direct probe of initial conditions and the most efficient way to look for non-Gaussianity of the type described by Eq. (1) [3]. Recent analyses of the angular bispectrum from 4-year COBE data [10] yield a weak upper limit, $|f_{\rm NL}| < 1.5 \times 10^3$. The analysis of the diagonal angular bispectrum of the Maxima dataset [11] also provides a very weak constraint: $|f_{\rm NL}| < 2330$. According to Komatsu and Spergel [4], the minimum value of $|f_{\rm NL}|$ that will become detectable from the analysis of MAP and *Planck* data, after properly subtracting detector noise and foreground contamination, is as large as $\sim$ 20, and 5 respectively.

In this paper we show that sizeable and detectable non-Gaussian perturbations both in the adiabatic and the isocurvature modes naturally arise during inflation when extra scalar degrees of freedom are present other than the inflaton field. In such a case, the adiabatic and isocurvature perturbations are correlated [12, 13, 14, 16] as a result of an oscillation mechanism similar to the phenomenon leading to neutrino oscillations [14] [1]. This may happen, for instance, if the inflaton field is coupled to the other scalar degrees of freedom, as expected on particle physics grounds. If these scalar degrees of freedom have large self-interactions, their quantum fluctuations are intrinsically non-Gaussian. This non-Gaussianity is transferred to the inflaton sector through the oscillation mechanism and is left imprinted in the adiabatic and isocurvature modes. We show that the CMB angular bispectrum is sourced not only by the intrinsic adiabatic and isocurvature bispectrum but also by cross-correlation terms, providing a characteristic and detectable signature of these non-Gaussian inflationary perturbations.

The idea that an isocurvature perturbation mode produced during inflation could

[1] This phenomenon was first described in Ref. [15] where it was pointed out that dangerous relics may be generated as coherent states through the oscillation mechanism with the inflaton field in the preheating phase after inflation, leading to tight constraint on the reheating temperature.

be non-Gaussian is certainly not new [17, 18, 19]. These scenarios, however, have the disadvantage that it is generally difficult to fit the observed pattern of CMB anisotropies in terms of isocurvature perturbations alone. The possibility of generically cross-correlating the adiabatic and isocurvature modes is attractive [20, 21, 22] both because of its wider capability of reproducing the observed CMB angular power-spectrum and because of the possibility of introducing non-Gaussianity in the adiabatic mode too. Moreover the characteristic signatures of these non-Gaussian inflationary perturbations could be also a way to break some degeneracies between the cosmological parameters which usually arise in scenarios where correlated adiabatic and isocurvature perturbations are present [21, 22]. Indeed there exist other mechanisms to produce non-Gaussian primordial perturbations, such as single-field models allowing for an initial state which is not the ground state [23] or, outside the inflationary paradigm, cosmic defects [24]. The scenario we propose differs from the other mechanisms since it can generate non-Gaussian perturbations which are adiabatic through the oscillation mechanism mentioned above.

The plan of the paper is as follows. In Section 2 we derive a general formula for the CMB angular bispectrum in models where correlated adiabatic and isocurvature perturbation modes are present. The physical mechanism by which these modes can be produced during inflation is summarized in Section 3. The resulting form for the angular bispectrum is obtained in Section 4 in the case of adiabatic plus cold dark matter isocurvature perturbations. Section 5 contains our conclusions.

2 The CMB Angular Bispectrum

In order to investigate possible non-Gaussian features of the CMB one can consider the angular three point correlation function

$$\left\langle \frac{\Delta T}{T}(\hat{\mathbf{n}}_1)\frac{\Delta T}{T}(\hat{\mathbf{n}}_2)\frac{\Delta T}{T}(\hat{\mathbf{n}}_3)\right\rangle = \sum_{l_i,m_i} \langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}\rangle Y_{l_1m_1}(\hat{\mathbf{n}}_1)Y_{l_2m_2}(\hat{\mathbf{n}}_2)Y_{l_3m_3}(\hat{\mathbf{n}}_3)\,, \quad (2)$$

where hats denote unit vectors and we have used the usual expansion of the CMB temperature anisotropy in spherical harmonics $Y_{lm}(\hat{\mathbf{n}})$ with coefficients

$$a_{lm} = \int d\hat{\mathbf{n}}\, Y^*_{lm}(\hat{\mathbf{n}}) \frac{\Delta T}{T}(\hat{\mathbf{n}})\,. \tag{3}$$

The angular CMB bispectrum is the harmonic conjugate of the three-point correlation function and is given by

$$\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} B_{l_1 l_2 l_3}, \tag{4}$$

where the first term is the Wigner 3j symbol and $B_{l_1 l_2 l_3}$ is the angle-averaged bispectrum, which is the observational quantity.

To calculate the bispectrum one has to properly take into account the initial conditions in the radiation dominated epoch after the end of inflation. Such initial conditions may reflect either the adiabatic or isocurvature nature of the cosmological perturbations. In general, however, one expects a mixture of adiabatic and isocurvature perturbations with a nonvanishing cross-correlation [12, 13, 14, 16, 20, 21, 22].

For *pure* adiabatic perturbations the harmonic coefficients a_{lm} are given by [25]

$$a_{lm} = 4\pi\,(-i)^l \int d^3k\, \hat{\Phi}(\mathbf{k})\, \Delta_l(k) Y^*_{lm}(\hat{\mathbf{k}})\,, \tag{5}$$

where $\hat{\Phi}(\mathbf{k})$ indicates the primordial gravitational potential perturbation and $\Delta_l(k)$ is the *radiation*, or *CMB, transfer function*. In the large scale limit, one recovers the Sachs-Wolfe effect

$$\frac{\Delta T}{T} = \frac{1}{3}\Phi \tag{6}$$

where Φ is the gravitational potential at recombination, by choosing $\Delta_l(k) = 1/3\, j_l[k(\tau_0 - \tau_{rec})]$, τ_0 being the conformal time at present and τ_{rec} the conformal time at recombination.

In the case of pure isocurvature perturbations one simply inserts the initial entropic perturbation $S(\mathbf{k})$ in Eq. (5) in place of the gravitational potential perturbation (see, for example, [26]). Of course, this corresponds to a different radiation transfer function which can be called $\Delta_l^S(k)$.

Having the expression for a_{lm} it is possible to calculate the bispectrum. Following the formalism of Ref. [6], one finds for pure adiabatic perturbations the following expression

$$\begin{aligned}
\langle a_{l_1m_1} a_{l_2m_2} a_{l_3m_3} \rangle &= (4\pi)^3 (-i)^{l_1+l_2+l_3} \int d^3k_1 d^3k_2 d^3k_3 \\
&\times Y^*_{l_1m_1}(\hat{\mathbf{k}}_1) Y^*_{l_2m_2}(\hat{\mathbf{k}}_2) Y^*_{l_3m_3}(\hat{\mathbf{k}}_3) \\
&\times \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P^{(3)}_{\hat{\Phi}}(k_1, k_2, k_3) \\
&\times \Delta_{l_1}(k_1) \Delta_{l_2}(k_2) \Delta_{l_3}(k_3)
\end{aligned} \tag{7}$$

where

$$\langle \hat{\Phi}(\mathbf{k}_1) \hat{\Phi}(\mathbf{k}_2) \hat{\Phi}(\mathbf{k}_3) \rangle = \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P^{(3)}_{\hat{\Phi}}(k_1, k_2, k_3) \tag{8}$$

is the three-dimensional bispectrum of the gravitational potential. A similar expression holds for pure isocurvature perturbations. We now analyze what happens in the most general case in which both adiabatic and isocurvature modes are present and are correlated.

2.1 Mixture of adiabatic and entropy perturbations

In the case of initial adiabatic *plus* entropy perturbations, we write the coefficient a_{lm} as

$$a_{lm} = 4\pi\, (-i)^l \int d^3k \left[\hat{\Phi}(\mathbf{k})\, \Delta_l^A(k) + S(\mathbf{k})\, \Delta_l^S(k) \right] Y^*_{lm}(\hat{\mathbf{k}})\,, \tag{9}$$

where $\Delta_l^A(k)$ and $\Delta_l^S(k)$ are the tranfer functions for the adiabatic and the entropy perturbation modes, respectively. This expression is consistent with the fact that the equations for the evolution of cosmological perturbations are linear. As a check, one can consider the Sachs-Wolfe effect for adiabatic ($\hat{\Phi}$) plus cold dark matter isocurvature (S_c) perturbations [20]:

$$\begin{aligned}
\left(\frac{\Delta T}{T} \right)_{SW} &= \left(\frac{\Delta T}{T} \right)_{AD} + \left(\frac{\Delta T}{T} \right)_{ISOC} \\
&= \frac{1}{3}\, \Phi_A + 2\, \Phi_S\,.
\end{aligned} \tag{10}$$

The first term on the r.h.s. of Eq. (10), containing the gravitational potential at large scales ($k \ll aH$) at the time of recombination, corresponds to the case of pure adiabatic perturbations. The second term corresponds to the case of pure isocurvature perturbations. The two potentials are given by

$$\Phi_A = \frac{3}{10}\left(3 + \frac{4}{5}\Omega_\nu^{RD}\right)\hat{\Phi}\,, \qquad \Phi_S = -\frac{1}{5}\,\Omega_c^{MD}\,S_c\,, \tag{11}$$

where Ω_c^{MD} is the density parameter for the cold dark matter during the matter era and Ω_ν^{RD} the one for neutrinos during the radiation era. One can recover this result from Eq. (9) with the transfer functions $\Delta_l^A(k) = 1/3\, j_l(k\chi)$ and $\Delta_l^S(k) = -2/5\, j_l(k\chi)\Omega_c^{MD}$. Note that the full transfer functions take into account all the other effects playing a role in the generation of the temperature anisotropies $\Delta T/T$ (such as the integrated Sachs-Wolfe effect emerging – for example – in the presence of a cosmological constant, and various small scale effects [4]).

Given the expression (9), if adiabatic and entropy perturbations are correlated we find for the bispectrum a result similar to Eq. (7), but with a more complicated structure

$$\begin{aligned}
\langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}\rangle \;=\;& (4\pi)^3(-i)^{l_1+l_2+l_3}\int d^3k_1 d^3k_2 d^3k_3 \\
&\times Y^*_{l_1m_1}(\hat{\mathbf{k}}_1)Y^*_{l_2m_2}(\hat{\mathbf{k}}_2)Y^*_{l_3m_3}(\hat{\mathbf{k}}_3)\times\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \\
&\times[P^{(3)}_{\hat{\Phi}}(k_1,k_2,k_3)\,\Delta^A_{l_1}(k_1)\Delta^A_{l_2}(k_2)\Delta^A_{l_3}(k_3) \\
&+P^{(3)}_S(k_1,k_2,k_3)\,\Delta^S_{l_1}(k_1)\Delta^S_{l_2}(k_2)\Delta^S_{l_3}(k_3) \\
&+P^{(3)}_{AAS}(k_1,k_2,k_3)\,\Delta^A_{l_1}(k_1)\Delta^A_{l_2}(k_2)\Delta^S_{l_3}(k_3) \\
&+(A,S,A)+(S,A,A)+(S,S,A)+(S,A,S)+(A,S,S)]\,.
\end{aligned} \tag{12}$$

As expected, the bispectrum gets contributions from the adiabatic modes (8), from the isocurvature modes

$$\langle S(\mathbf{k}_1)S(\mathbf{k}_2)S(\mathbf{k}_3)\rangle = \delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)P^{(3)}_S(k_1,k_2,k_3) \tag{13}$$

and from the terms parametrizing the cross-correlation between adiabatic and isocurvature modes, for example

$$\langle\hat{\Phi}(\mathbf{k}_1)S(\mathbf{k}_2)\hat{\Phi}(\mathbf{k}_3)\rangle = \delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)P^{(3)}_{ASA}(k_1,k_2,k_3) \tag{14}$$

where we have adopted the notation

$$(A,S,A) \equiv P^{(3)}_{ASA}(k_1,k_2,k_3)\,\Delta^A_{l_1}(k_1)\Delta^S_{l_2}(k_2)\Delta^A_{l_3}(k_3)\,. \tag{15}$$

Performing the angular integration following Ref. [6], we obtain Eq. (4), where

$$\begin{aligned} B_{l_1l_2l_3} &= (8\pi)^3\sqrt{\frac{(2l_1+1)(2l_2+1)(2l_3+1)}{4\pi}}\begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix} \\ &\times \int dk_1\,k_1^2\,dk_2\,k_2^2\,dk_3\,k_3^2\,J_{l_1l_2l_3}(k_1,k_2,k_3)\times \\ &\quad [P^{(3)}_{\hat{\Phi}}(k_1,k_2,k_3)\,\Delta^A_{l_1}(k_1)\Delta^A_{l_2}(k_2)\Delta^A_{l_3}(k_3)\;+ \\ &\quad P^{(3)}_{S}(k_1,k_2,k_3)\,\Delta^S_{l_1}(k_1)\Delta^S_{l_2}(k_2)\Delta^S_{l_3}(k_3)\;+ \\ &\quad P^{(3)}_{AAS}(k_1,k_2,k_3)\,\Delta^A_{l_1}(k_1)\Delta^A_{l_2}(k_2)\Delta^S_{l_3}(k_3)\;+ \\ &\quad (A,S,A)+(S,A,A)+(S,S,A)+(S,A,S)+(A,S,S)]\,. \end{aligned} \tag{16}$$

Note that the integral in Eq. (16) is proportional to the *reduced* bispectrum defined in Ref. [4]. Indeed it contains all the physical information on the bispectrum.

Our goal is now to show that large contributions to the bispectrum (12) may naturally arise when adiabatic and isocurvature modes are correlated.

3 Adiabatic and entropy perturbations from inflation

Correlated adiabatic and isocurvature modes can be generated during a period of inflation in which several scalar fields are present [12, 13, 14, 16]. Indeed, adiabatic (curvature) perturbations are produced during a period of cosmological inflation that is driven by a single scalar field, the inflaton. On particle physics grounds – though – it is natural to expect that this scalar field is coupled to other scalar degrees of freedom. This gives rise to oscillations between the perturbation of the inflaton field and the perturbations of the other scalar degrees of freedom, similar to the phenomenon of neutrino oscillations. The crucial observation is that – since the degree of mixing is governed by the squared mass matrix of the scalar fields – the oscillations can occur even if the energy density of the extra scalar fields is much smaller than the energy density of the inflaton. The probability of oscillation is resonantly amplified when perturbations cross the horizon and the perturbations in the inflaton field may disappear at horizon crossing giving rise to perturbations in scalar fields other than the inflaton. Adiabatic and isocurvature perturbations are inevitably correlated at the end of inflation [14, 16].

It is exactly this strong correlation which may give rise to large non-Gaussian features in the CMB anisotropy spectrum. This is a simple, but important point. Gaussian features in the CMB anisotropies are usually expected in inflationary models because the inflaton potential is required to be very flat. This amounts to saying that the interaction terms in the inflaton potential are present, but small and non-Gaussian features are suppressed since the non-linearities in the inflaton potential are suppressed too. On the other hand, nothing prevents the inflaton field from being coupled to another scalar degree of freedom whose energy density is much smaller than the one stored in the inflaton field. It is natural to expect that the the self-interactions of such extra field or the interaction terms with the inflaton field are sizeable and they represent potential non-linear sources for non-Gaussianity. If during the inflationary epoch, oscillations between the perturbation of the inflaton field and the perturbations of the other scalar degrees of freedom occur, the non-Gaussian features generated in the system of the extra field are efficiently communicated to the inflaton sector and may be left imprinted in the CMB anisotropies.

Let us consider for simplicity the case of two scalar fields ϕ and χ interacting through a generic potential $V(\phi, \chi)$. The study of the field fluctuations $\delta\phi$ and $\delta\chi$ can be done using the Sasaki-Mukhanov variables[2] [27]

$$Q_I \equiv \delta\phi_I + \frac{\dot{\phi}_I}{H}\psi \tag{17}$$

where $I = 1, 2$ with $\delta\phi_1 = \delta\phi$, $\delta\phi_2 = \delta\chi$ and ψ is the linear perturbation in the line element of the metric

$$ds^2 = -(1+2A)dt^2 + 2aB_i dx^i dt + a^2[(1-2\psi)\delta_{ij} + 2E_{ij}]dx^i dx^j. \tag{18}$$

Using such variables it is possible to define the adiabatic and entropy fields Q_A and δs in terms of the original field perturbations Q_ϕ and Q_χ [13]

$$Q_A = (\cos\beta)Q_\phi + (\sin\beta)Q_\chi\,, \tag{19}$$

$$\delta s = (\cos\beta)Q_\chi - (\sin\beta)Q_\phi\,, \tag{20}$$

where

$$\cos\beta \equiv c_\beta = \frac{\dot{\phi}}{\sqrt{\dot{\phi}^2 + \dot{\chi}^2}}\,, \qquad \sin\beta \equiv s_\beta = \frac{\dot{\chi}}{\sqrt{\dot{\phi}^2 + \dot{\chi}^2}}\,, \tag{21}$$

and the dots stand for the derivatives with respect to the cosmic time t.

The curvature perturbation [28]

$$\mathcal{R} = H \sum_I \left(\frac{\dot{\varphi}_I}{\sum_{J=1}^N \dot{\varphi}_J^2} \right) Q_I \tag{22}$$

deep in the radiation era can be written in terms of the adiabatic field Q_A

$$\mathcal{R}_{rad} = \frac{H}{c_\beta\dot{\phi} + s_\beta\dot{\chi}} Q_A \tag{23}$$

where the r.h.s of this equation is evaluated at the end of inflation.

Let us now introduce the slow-roll parameters for the two scalar fields ϕ and χ

$$\epsilon_I = \frac{M_{Pl}^2}{16\pi}\left(\frac{V_{\phi_I}}{V}\right)^2 \quad \text{and} \quad \eta_{IJ} = \frac{M_{Pl}^2}{8\pi}\frac{V_{\phi_I\phi_J}}{V}\,, \tag{24}$$

where M_{Pl} is the Planck mass, $V_{\phi_I} = \partial V/\partial\phi_I$, and $\phi_I = \phi, \chi$.

Having a sucessfull period of inflation requires that the potential is flat enough, that

[2] To simplify the calculation of the three-point correlation functions one can reduce to a particular gauge, such as the spatially flat gauge ($\psi = 0$) in which the Q_I variables concide with $\delta\phi_I$.

is ϵ_I and $|\eta_{IJ}| \ll 1$. Now, making an expansion in the slow roll parameters to lowest order, it is possibile to write the gravitational potential $\hat{\Phi}$ as [14, 16]

$$\hat{\Phi} = \frac{2}{3}\mathcal{R}_{rad} = \frac{2}{3}\frac{\sqrt{4\pi}}{M_{Pl}}\frac{1}{\sqrt{\epsilon_{tot}}}\,Q_A \tag{25}$$

where $\epsilon_{tot} = \epsilon_\phi + \epsilon_\chi$.

Under the hypothesis that the scalar field ϕ decays into "ordinary" matter (the present day photons, neutrinos and baryons), while the scalar field χ decays only into cold dark matter (or it does not decay at all, like in the case of the superheavy dark matter [30]), an adiabatic ($\hat{\Phi}$) and a cold dark matter isocurvature mode (S_c) will be generated in the post-inflationary epoch. To lowest order in the slow roll parameters, they are given by expression (25) and [14, 16]

$$S_c = -3\,\frac{\sqrt{4\pi}}{m_{Pl}}\,\frac{\sqrt{\epsilon_{tot}}}{(\pm\sqrt{\epsilon_\phi})(\pm\sqrt{\epsilon_\chi})}\delta s \tag{26}$$

where the r.h.s. is evaluated at the end of inflation as a matching condition. In a short-hand notation we can write

$$\hat{\Phi} = A_0\, Q_A, \qquad S = S_0\, \delta s, \tag{27}$$

where A_0 and S_0 are just the "amplitudes" of $\hat{\Phi}$ and S_c.

As we will show in the next section, since it is quite natural to expect a nonzero correlation between the adiabatic field Q_A and the entropy field δs generated during inflation [14], non-Gaussian features in the CMB anisotropies may be left imprinted.

4 Primordial non-Gaussianity from inflation

We are now in the position of relating the bispectrum in Eq. (12) with the expressions for $\hat{\Phi}$ and S_c originated during a a period of inflation.

Consider, for example, the $\langle\hat{\Phi}(\mathbf{k}_1)\hat{\Phi}(\mathbf{k}_2)\hat{\Phi}(\mathbf{k}_3)\rangle$ term. One finds

$$\langle\hat{\Phi}(\mathbf{k}_1)\hat{\Phi}(\mathbf{k}_2)\hat{\Phi}(\mathbf{k}_3)\rangle \;\; = \;\; A_0^3\langle Q_A(\mathbf{k}_1)Q_A(\mathbf{k}_2)Q_A(\mathbf{k}_3)\rangle$$

$$
\begin{aligned}
&= A_0^3\langle(c_\beta Q_{\phi 1}+s_\beta Q_{\chi 1})(c_\beta Q_{\phi 2}+s_\beta Q_{\chi 2})(c_\beta Q_{\phi 3}+s_\beta Q_{\chi 3})\rangle \qquad (28)\\
&= A_0^3\,[c_\beta^3\,\langle Q_{\phi 1}Q_{\phi 2}Q_{\phi 3}\rangle + c_\beta^2 s_\beta\langle Q_{\phi 1}Q_{\phi 2}Q_{\chi 3}\rangle + c_\beta^2 s_\beta\langle Q_{\phi 1}Q_{\chi 2}Q_{\phi 3}\rangle\\
&\quad + c_\beta s_\beta^2\langle Q_{\phi 1}Q_{\chi 2}Q_{\chi 3}\rangle + s_\beta c_\beta^2\langle Q_{\chi 1}Q_{\phi 2}Q_{\phi 3}\rangle + s_\beta^2 c_\beta\langle Q_{\chi 1}Q_{\phi 2}Q_{\chi 3}\rangle\\
&\quad + s_\beta^2 c_\beta\langle Q_{\chi 1}Q_{\chi 2}Q_{\phi 3}\rangle + s_\beta^3\langle Q_{\chi 1}Q_{\chi 2}Q_{\chi 3}\rangle]
\end{aligned}
$$

where, for example, $Q_{\phi 1}$ stands for $Q_\phi(\mathbf{k}_1)$ and we have used Eq. (19). Analogous expressions hold for the remaining terms.

Our goal is now to show that a large amount of non-Gaussianity can be generated in the presence of correlated adiabatic and entropy perturbations. First of all, we note that the bispectrum is a sum of different three-point correlation functions. The coefficients in front of each correlation function involve mixing angles which parametrize the amount of mixing between the adiabatic and the isocurvature modes. If such mixing is sizeable, all coefficients are of order unity and one expects that nonlinearities in the perturbation of the scalar field χ may be efficiently transferred to the inflaton sector, thus generating large non-Gaussian features.

Because expressions are quite lengthy and might obscure our point, from now on and just for illustrative purposes we make some simplifying hypothesis and assume that the dominant nonlinear terms are those sourced by the three-point correlation function $\langle Q_{\chi 1}Q_{\chi 2}Q_{\chi 3}\rangle$. This could be the case for a Langrangian of scalar fields ϕ and χ in which the largest coupling is for the $\frac{\mu}{3}\chi^3$-term. Let us also assume that the field χ is lighter than the Hubble rate during inflation.

Under these assumptions, we can rewrite the bispectrum (12) as

$$
\begin{aligned}
\langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}\rangle &= (4\pi)^3(-i)^{l_1+l_2+l_3}\int d^3k_1 d^3k_2 d^3k_3 \qquad (29)\\
&\times Y^*_{l_1m_1}(\hat{\mathbf{k}}_1)Y^*_{l_2m_2}(\hat{\mathbf{k}}_2)Y^*_{l_3m_3}(\hat{\mathbf{k}}_3)\times\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\\
&\times\{A_0^3 s_\beta^3\,P^{(3)}_{Q_\chi}(k_1,k_2,k_3)\Delta^A_{l_1}(k_1)\Delta^A_{l_2}(k_2)\Delta^A_{l_3}(k_3)\\
&+S_0^3 c_\beta^3\,P^{(3)}_{Q_\chi}(k_1,k_2,k_3)\Delta^S_{l_1}(k_1)\Delta^S_{l_2}(k_2)\Delta^S_{l_3}(k_3)\\
&+A_0^2 S_0 s_\beta^2 c_\beta\,P^{(3)}_{Q_\chi}(k_1,k_2,k_3)[\Delta^A_{l_1}(k_1)\Delta^A_{l_2}(k_2)\Delta^S_{l_3}(k_3)
\end{aligned}
$$

$$+(k_1 \leftrightarrow k_3; l_1 \leftrightarrow l_3) + (k_2 \leftrightarrow k_3; l_2 \leftrightarrow l_3)]$$

$$+S_0^2 A_0 c_\beta^2 s_\beta \, P^{(3)}_{Q_\chi}(k_1, k_2, k_3)[\Delta^S_{l_1}(k_1)\Delta^S_{l_2}(k_2)\Delta^A_{l_3}(k_3)$$

$$+(k_1 \leftrightarrow k_3; l_1 \leftrightarrow l_3) + (k_2 \leftrightarrow k_3; l_2 \leftrightarrow l_3)]\,\}$$

where

$$\langle Q_{\chi 1} Q_{\chi 2} Q_{\chi 3} \rangle = \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P^{(3)}_{Q_\chi}(k_1, k_2, k_3)\,. \tag{30}$$

The angular part of the integral can be calculated as done in subsection 2.1.

The next step is to further reduce the expression for the bispectrum by expanding $\langle Q_{\chi 1} Q_{\chi 2} Q_{\chi 3} \rangle$. This is necessary because, in the presence of large mixing, Q_ϕ and Q_χ are not "mass-eigenstates" of the system, but just interaction eigenstates. The situation here is analogous to what happens for light neutrinos where the three different flavors of neutrinos represent interaction eigenstates, but they do not represent mass-eigenstates because of the mixing among the flavors giving rise to the phenomenon of neutrino oscillations.

We first define the comoving fields $\tilde{Q}_\phi = aQ_\phi$ and $\tilde{Q}_\chi = aQ_\chi$ and then we introduce a basis for annihilation and creation operators a_i and $a_i^\dagger$ [14]. We perform the decomposition (τ is the conformal time):

$$\begin{aligned}
\begin{pmatrix} \widetilde{Q}_\phi \\ \widetilde{Q}_\chi \end{pmatrix} &= \mathcal{U} \int \frac{d^3k}{(2\pi)^{3/2}} \left[e^{i\mathbf{k}\cdot\mathbf{x}}\, h(\tau) \begin{pmatrix} a_1(k) \\ a_2(k) \end{pmatrix} + \text{h.c.} \right], \\
\begin{pmatrix} \Pi_{\widetilde{Q}_\phi} \\ \Pi_{\widetilde{Q}_\chi} \end{pmatrix} &= \mathcal{U} \int \frac{d^3k}{(2\pi)^{3/2}} \left[e^{i\mathbf{k}\cdot\mathbf{x}}\, \widetilde{h}(\tau) \begin{pmatrix} a_1(k) \\ a_2(k) \end{pmatrix} + \text{h.c.} \right],
\end{aligned} \tag{31}$$

where $\Pi_{\widetilde{Q}_\phi}$ and $\Pi_{\widetilde{Q}_\chi}$ are the conjugate momenta of $\widetilde{Q}_\phi$ and $\widetilde{Q}_\chi$ respectively, and h and $\widetilde{h}$ are two 2×2 matrices satisfying the relation

$$\left[h\, \widetilde{h}^* - h^*\, \widetilde{h}^T \right]_{ij} = i\,\delta_{ij}, \tag{32}$$

derived from the canonical quantization condition.

The matrix $\mathcal{U}$ is a rotation matrix

$$\mathcal{U} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{33}$$

which diagonalizes the squared mass matrix of the two scalar field perturbations Q_ϕ and Q_χ

$$\mathcal{M}^2_{IJ} = V_{\phi_I\phi_J} - 8\pi/M^2_{Pl}a^3 \left(a^3/H \, \dot{\phi}_I\dot{\phi}_J\right)^{\cdot} \simeq \frac{8\pi V}{M^2_{Pl}}\left[\eta_{IJ} - 2\,(\pm\sqrt{\epsilon_I})(\pm\sqrt{\epsilon_J})\right] , \qquad (34)$$

where the sign $\pm$ stands for the cases $\dot{\phi}_I(\dot{\phi}_J) > 0$ and < 0 respectively. The mixing angle θ is given by

$$\tan 2\theta = \frac{2\,\mathcal{M}^2_{\chi\phi}}{\mathcal{M}^2_{\phi\phi} - \mathcal{M}^2_{\chi\chi}} \, . \qquad (35)$$

One can envisage different situations:

i) Inflation is driven by the inflaton field ϕ and there is another scalar field χ with a simple polynomial potential $V(\chi) \propto \chi^n$ leading to a vacuum expectation value $\langle\chi\rangle = 0$. In such a case, $\sin\beta = \sin\theta = 0$ and there is no mixing between the inflaton field and the χ-field as well as no cross-correlation between the adiabatic and isocurvature modes. Nonvanishing non-Gaussianity will be present in the isocurvature mode. This is indeed a known result [17, 19]. Non-Gaussian adiabatic perturbations may also arise if the χ-field decays late after inflation [19, 29].

ii) Inflation is driven by two scalar fields ϕ and χ with equal mass, $V = \frac{m^2}{2}(\phi^2+\chi^2)$. In such a case the mixing is maximal, $\beta = \theta = \pi/4$. Nevertheless, the cross-correlation is again vanishing [13, 14, 16] and the bispecrum gets contributions from adiabatic and isocurvature modes independently, since in this case the terms parametrizing the cross-correlation in Eq. (12) vanish. A term $\frac{\mu}{3}\chi^3$ in the Lagrangian would be a source of non-Gaussianity and at the same time it would switch on a cross correlation between the adiabatic and the isocurvature modes, thus producing nonzero cross terms in Eq. (29). However, these non-Gaussianities would be small because of slow-roll conditions.

iii) Inflation is driven by an inflaton field ϕ and there is another scalar field χ whose vacuum expectation value depends on the inflaton field and – eventually – on the Hubble parameter H and some other mass scale μ, $\langle\chi\rangle = f(\phi, H, \mu)$. Under these circumstances, $\langle\dot{\chi}\rangle = \partial f/\partial\phi\,\dot{\phi} + \partial f/\partial H\,\dot{H}$. As in illustrative case, let us restrict ourselves to the case in which $\partial f/\partial\phi\,\dot{\phi}$ is the dominant term and we can approximate $\langle\dot{\chi}\rangle = \partial f/\partial\phi\dot{\phi}$. We have therefore $\tan\beta \simeq \partial f/\partial\phi$ and $\dot\beta \simeq (\partial f/\partial\phi)^{\cdot}/[1+(\partial f/\partial\phi)^2]$. In such a case, cross-correlation between the adiabatic and the isocurvature modes may be large and non-Gaussianity may be efficiently transferred from one mode to the other.

We can now reduce $\langle Q_{\chi 1} Q_{\chi 2} Q_{\chi 3}\rangle$ using the decomposition (31) and making some further approximations justified if slow-roll conditions are attained. In fact, using a perturbative method, it can be checked that the contributions to $\langle Q_{\chi 1} Q_{\chi 2} Q_{\chi 3}\rangle$ coming from terms proportional to the non-diagonal elements h_{12} and h_{21} can be neglected since h_{12} and h_{21} are $\mathcal{O}(\epsilon_I, \eta_{IJ})$, and on superhorizon scales $k \ll aH$ h_{11} and h_{22} are Hankel functions [14]. Thus we can neglect the non diagonal terms of the h matrix and write[3]

$$\begin{aligned} \tilde{Q}_\chi(\mathbf{k}) &= [s_\theta h_{11}\, a_1(\mathbf{k}) + c_\theta h_{22}\, a_2(\mathbf{k})] + [s_\theta h_{11}^*\, a_1^\dagger(-\mathbf{k}) + c_\theta h_{22}^*\, a_2^\dagger(-\mathbf{k})] \\ &= s_\theta [h_{11}\, a_1(\mathbf{k}) + h_{11}^*\, a_1^\dagger(-\mathbf{k})] + c_\theta [h_{22}\, a_2(\mathbf{k}) + h_{22}^*\, a_2^\dagger(-\mathbf{k})] \\ &\equiv s_\theta h_1\, I_1 + c_\theta h_2\, I_2\,, \end{aligned} \tag{36}$$

where, for simplicity of notation, we have indicated the diagonal terms of the h matrix as $h_{11} \equiv h_1\,, h_{22} \equiv h_2$ and we have defined two new fields I_1 and I_2 just by collecting the functions h_{11} and h_{22}. After all these manipulations we arrive at three-point function

$$\begin{aligned} \langle \tilde{Q}_{\chi 1} \tilde{Q}_{\chi 2} \tilde{Q}_{\chi 3}\rangle &= \langle (s_\theta\, h_{1\mathbf{1}}\, I_{1\mathbf{1}} + c_\theta\, h_{2\mathbf{1}}\, I_{2\mathbf{1}})(s_\theta\, h_{1\mathbf{2}}\, I_{1\mathbf{2}} + c_\theta\, h_{2\mathbf{2}}\, I_{2\mathbf{2}})(s_\theta\, h_{1\mathbf{3}}\, I_{1\mathbf{3}} + c_\theta\, h_{2\mathbf{3}}\, I_{2\mathbf{3}})\rangle \\ &= s_\theta^3\, (h_{1\mathbf{1}}\, h_{1\mathbf{2}}\, h_{1\mathbf{3}}) \langle I_{1\mathbf{1}}\, I_{1\mathbf{2}}\, I_{1\mathbf{3}}\rangle + s_\theta^2 c_\theta\, (h_{1\mathbf{1}}\, h_{1\mathbf{2}}\, h_{2\mathbf{3}}) \langle I_{1\mathbf{1}}\, I_{1\mathbf{2}}\, I_{2\mathbf{3}}\rangle \\ &\quad + s_\theta^2 c_\theta\, (h_{1\mathbf{1}}\, h_{2\mathbf{2}}\, h_{1\mathbf{3}}) \langle I_{1\mathbf{1}}\, I_{2\mathbf{2}}\, I_{1\mathbf{3}}\rangle + s_\theta c_\theta^2\, (h_{1\mathbf{1}}\, h_{2\mathbf{2}}\, h_{2\mathbf{3}}) \langle I_{1\mathbf{1}}\, I_{2\mathbf{2}}\, I_{2\mathbf{3}}\rangle \\ &\quad + c_\theta s_\theta^2\, (h_{2\mathbf{1}}\, h_{1\mathbf{2}}\, h_{1\mathbf{3}}) \langle I_{2\mathbf{1}}, I_{1\mathbf{2}}\, I_{1\mathbf{3}}\rangle + c_\theta^2 s_\theta\, (h_{2\mathbf{1}}\, h_{1\mathbf{2}} h_{2\mathbf{3}}) \langle I_{2\mathbf{1}}\, I_{1\mathbf{2}}\, I_{2\mathbf{3}}\rangle \\ &\quad + c_\theta^2 s_\theta\, (h_{2\mathbf{1}}\, h_{2\mathbf{2}}\, h_{1\mathbf{3}}) \langle I_{2\mathbf{1}}\, I_{2\mathbf{2}}\, I_{1\mathbf{3}}\rangle + c_\theta^3\, (h_{2\mathbf{1}}\, h_{2\mathbf{2}}\, h_{2\mathbf{3}}) \langle I_{2\mathbf{1}}\, I_{2\mathbf{2}}\, I_{2\mathbf{3}}\rangle\,, \end{aligned} \tag{37}$$

where the indices " $\mathbf{1}, \mathbf{2}, \mathbf{3}$ ", as usual, indicate that the quantities are evaluated at $\mathbf{k}_1, \mathbf{k}_2$ and $\mathbf{k}_3$.

If we indicate

$$\langle I_{i\mathbf{1}}\, I_{j\mathbf{2}}\, I_{k\mathbf{3}}\rangle = \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\, P^3_{ijk}(k_1, k_2, k_3),\;\; i,j,k = 1,2 \tag{38}$$

[3] In such a case the system is diagonalized by the matrix $\mathcal{U}$.

because of rotation and translation invariance, it is easy to convince oneself that the terms $\langle I_{i\mathbf{1}}\, I_{j\mathbf{2}}\, I_{k\mathbf{3}}\rangle$ are invariant under an exchange of $\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3$. Thus, taking into account that the operators I_1 and I_2 commute, one can check that, for example, $\langle I_{1\mathbf{1}}\, I_{1\mathbf{2}}\, I_{2\mathbf{3}}\rangle$ is equal to $\langle I_{1\mathbf{1}}\, I_{2\mathbf{2}}\, I_{1\mathbf{3}}\rangle$. The expression (37) is thus further simplified to

$$\begin{aligned}\langle \tilde{Q}_{\chi 1}\tilde{Q}_{\chi 2}\tilde{Q}_{\chi 3}\rangle \;&=\; s_\theta^3\,(h_{1\mathbf{1}}\,h_{1\mathbf{2}}\,h_{1\mathbf{3}})\langle I_{1\mathbf{1}}\,I_{1\mathbf{2}}\,I_{1\mathbf{3}}\rangle + c_\theta^3\,(h_{2\mathbf{1}}\,h_{2\mathbf{2}}\,h_{2\mathbf{3}})\langle I_{2\mathbf{1}}\,I_{2\mathbf{2}}\,I_{2\mathbf{3}}\rangle \\ &\quad + s_\theta^2 c_\theta\,\langle I_{1\mathbf{1}}\,I_{1\mathbf{2}}\,I_{2\mathbf{3}}\rangle(h_{1\mathbf{1}}\,h_{1\mathbf{2}}\,h_{2\mathbf{3}} + h_{1\mathbf{1}}\,h_{2\mathbf{2}}\,h_{1\mathbf{3}} + h_{2\mathbf{1}}\,h_{1\mathbf{2}}\,h_{1\mathbf{3}}) \\ &\quad + c_\theta^2 s_\theta\,\langle I_{1\mathbf{1}}\,I_{2\mathbf{2}}\,I_{2\mathbf{3}}\rangle\;(h_{1\mathbf{1}}\,h_{2\mathbf{2}}, h_{2\mathbf{3}} + h_{2\mathbf{1}}\,h_{1\mathbf{2}}\,h_{2\mathbf{3}} + h_{2\mathbf{1}}\,h_{2\mathbf{2}}\,h_{1\mathbf{3}})\,. \end{aligned} \tag{39}$$

In order to make a quantitative estimate of the three-point function (39), we first notice that the coefficients in front of the various terms are of order unity, provided the degree of mixing is large. We can borrow the expression for each three-point function appearing in Eq. (39) from the calculation of [2], which is done for an effectively massless scalar field χ with cubic self-interactions. In the de Sitter background it is given by

$$\langle(\chi(\mathbf{k}_1)\chi(\mathbf{k}_2)\chi(\mathbf{k}_3)\rangle = \frac{1}{6}\mu H^2 (k_1k_2k_3)^{-3} F(k_1,k_2,k_3)\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\;, \tag{40}$$

where

$$F(k_1,k_2,k_3) \simeq -\beta(k_1^3+k_2^3+k_3^3) \tag{41}$$

and, for instance, $\beta \sim 60$ if one is interested in the scales relevant for large-angle CMB anisotropies. Notice that the functional form of the bispectrum is the same found by Gangui et al. [5], who made different assumptions and used the stochastic approach to inflation.

Plugging the above expressions into the CMB angular bispectrum one gets the standard relation [5, 3] giving the angular bispectrum $B_{l_1l_2l_3}$ as a sum of products of two angular power-spectra, $C_{l_i}C_{l_j}$. The non-Gaussianity amplitude is monitored by the dimensionless strength $f_{\rm NL} = \mathcal{O}(\mu/H)$.

5 Conclusions

In this paper we have studied inflationary models where extra scalar degrees of freedom other than the inflaton exist. This allows isocurvature perturbation modes to be switched on during the inflationary evolution besides the usual adiabatic one. As previously shown [13, 14, 16], a generic prediction of these models is that non-zero cross-correlations arise among adiabatic and isocurvature fluctuations. Here we exploited this physical process as an efficient tool to transfer non-Gaussian features from the isocurvature to the adiabatic mode. Sizeable non-Gaussianity can be easily produced in the non-inflatonic sector, by self-interactions leading to non-linearities in their evolution. This is because, unlike the inflaton case, the self-interaction strength in such an extra scalar sector does not suffer from the usual slow-roll conditions. In order to make use of our results for practical purposes, one might introduce a simple non-Gaussian model. For instance, one can parametrize the non-Gaussianity in the peculiar gravitational potential as

$$\Phi = \varphi_1 + f_{\rm NL}(\varphi_2^2 - \langle\varphi_2^2\rangle) + \mathcal{O}(f_{\rm NL}^2) \qquad (42)$$

(and a similar expression for the entropy mode), where φ_1 and φ_2 are zero-mean Gaussian fields with non-zero cross-correlation $\langle\varphi_1\varphi_2\rangle \neq 0$. The non-Gaussianity strength $f_{\rm NL}$, being sourced by the non-inflatonic scalar sector of the theory, is not generally constrained by the slow-roll conditions of inflationary dynamics. This may make the non-Gaussian signatures accessible by future CMB satellite experiments.

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